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MPC in urban traffic management

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1. Introduction

More and more people are concerned about the negative phenomenon resulted by the negative effects of the growing traffic motorization. Traffic congestion is the primary direct impact which became everyday occurrence in the last decade. As world trade is continuously increasing, it is obvious that congestions represent also a growing problem. The capacity of the traffic networks saturates during rush hours. At the same time, the traditional traffic management is getting less effective in sustaining a manageable traffic flow. Therefore, external impacts appear causing new costs for the societies. As a possible solution the predictive control based strategy can be applied. The chapter investigates the applicability of MPC strategy specialized in urban traffic management in order to relieve traffic congestion, to reduce travel time and to improve homogeneous traffic flow. Over the theory the realization of the control method is also presented. Firstly we give a historical summary of adaptive traffic control. The brief results of MPC and its related methods in urban traffic control are presented. Then we introduce the modeling possibilities of urban traffic as the appropriate model means an important aspect of the control design. The use of MPC requires a state space theory approach. Therefore the so called Store-and-forward model is chosen which can be directly translated to state space. We analyze the model in details showing the real meaning of system matrices. The constraints of urban traffic system is also discussed which heavily influence modeling and control. The next section presents the simulation environment which is used to demonstrate the developed control methods. Thereinafter we present the main results of MPC in traffic application. The idea to apply MPC in urban traffic network is induced by the fact that the distance is relatively short between several intersections with traffic lights. Hence it is advisable to coordinate the operation of the intersection controller devices. Several intersections are near to each other in smaller or bigger networks, primarily in cities, the coordination is especially emphasized. The development of new control strategies is a real demand of nowadays. One of the possible solutions is the practical application of MPC. The aim of the control is to increase capacity. To test and validate our control strategy we apply it to a real-word transportation network where the actual system is not efficient enough to manage the traffic in rush hours. The simulation results show the effectiveness of the control design. After the presentation of the practical urban traffic MPC the distributive solution of MPC has to be discussed. As the computational demand depends on the size of the network an efficient calculation method is

sought to solve online the MPC problem. The classical scheme for adaptive road traffic management structure is usually based on control center which processes and computes all signal control for the network. Another method for the control system architecture is the decentralized and distributed control scheme. This approach has numerous economical and technological advantages. Distributed traffic control is developed using iterative solution. The so-called Jacobi iteration algorithm is an efficient method to solve constrained and nonlinear programming problem which the original problem can be transformed for. An additional feature of the developed strategy is the ability to manage priority. If a preferred vehicle arrives to any junction of the network it will be automatically indicated. Its stage will be handled with priority getting maximum green time as possible in every cycle until the vehicle will not leave the intersection. It means practically that the cost function is dynamically modified by the system weights depending of presence of any preferred vehicles. Finally we would like to introduce the robust MPC problem in traffic management as our future work. The robustness of the traffic management means that even with the presence of some disturbances the system is able to find optimal control solution. We discuss the modification of the traffic model introduced in third section since the chosen method requires a special model structure.

2. Brief historical summary of adaptive road traffic control

In case the distance is relatively short between several intersections with traffic lights it is advisable to co-ordinate the operation of the intersection controller devices. The coordination may include public transport devices and pedestrian traffic besides vehicles. Where several intersections are near to each other in smaller or bigger networks, primarily in cities, the coordination is especially emphasized.

In the 1970's a new control strategy appears in the road traffic management. Beside the already extant fixed-time and traffic-actuated strategies the traffic-adaptive control is invented. A traffic control system that continuously optimizes the signal plan according to the actual traffic load is called an adaptive traffic control system. The essence of the functioning is that the changes to the active signal plan parameters are automatically implemented in response to the current traffic demand as measured by a vehicle detection system. Such system can be used as local or network-wide control.

The appearance of the adaptivity induces new developments of traffic control methods. The first adaptive systems like SCOOT (Hunt et al., 1982) or SCATS (Lowrie, 1982) are based on heuristic optimization algorithms. In the 1980's new optimization methods are introduced based on rolling horizon optimization using dynamic programming. Some examples are OPAC (Gartner, 1983), PRODYN (Farges et al. 1983), and RHODES (Sen & Head, 1997).

In the middle of the 1990's the first control method is introduced which adopts results of the modern control theory. The TUC system (Diakaki et al., 1999) applies a multivariable regulator approach to calculate in real time the network splits, while cycle time and offsets are calculated by other parallel algorithms. The basic methodology employed for split control by TUC is the formulation of the urban traffic control problem as a linear-quadratic (LQ) optimal control problem. The advantage of LQ control is the simplicity of the required real-time calculations which is an important aspect in network-wide signal control. However the algorithm has a main disadvantage. LQ control is not able to manage constraints on the control input (its importance is discussed in the next section). Therefore a

posteriori application is needed to force the constraints which may lead to suboptimal solution.

In the early 2000's the first results are published in the subject of MPC based traffic control. However these publications (e.g. Bellemans et al., 2002; Hegyi et al., 2003) are related to ramp metering and variable speed limit control of the freeway traffic management. MPC based urban traffic control approach is published by Tettamanti et al. (2008). The paper consists theory, realization and a real-word example. The main result is the possibility to overcome the disadvantage of the LQ problem mentioned above as the MPC method can take the constraints into consideration. These results constitute the basis of the chapter. The paper of Aboudolas et al. (2009) is published investigating large-scale traffic control problem and introducing the open-loop quadratic-programming control (QPC) as a possible method for optimal traffic management. The paper concludes that for the application of the QPC methodology in real time, the corresponding algorithms may be embedded in a rolling-horizon (model-predictive) scheme which constitutes the part of future works.

In 2010 as a development result of Tettamanti et al. (2008) the paper of Tettamanti & Varga (2010) is published which introduces a distributed realization of an MPC based traffic control system. The publication's results will be also enlightened in detail in the chapter.

3. Urban traffic modeling

Modeling and control are coherent notions in control theory as the model highly determines the applicable methods for control. In the previous chapter various control approaches were presented. All of them use an appropriate traffic modeling technique for functioning. Apparently, the modern control theory based traffic management strategies apply the state space approach. The state space modeling is derived from the so called Store-and-forward model (Gazis & Potts, 1963) which introduces a model simplification that enables the mathematical description of the traffic flow process. This modeling technique opens the way to the application of a number of highly efficient optimization methods such as LQ control, MPC, or robust LMI based control. Before to begin to investigate the MPC based traffic control the properties of the model have to be discussed in detail.

3.1 From Store-and-forward traffic modeling to state space representation

The following derivation of the state space model reflects the results of the paper of Diakaki et al. (1999).

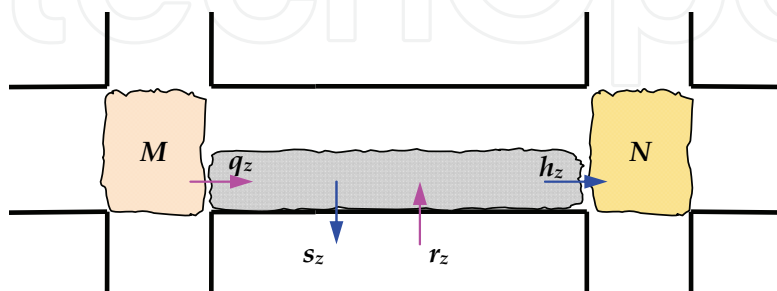


Fig. 1. The Store-and-forward traffic model

The two basic parts of an urban road traffic network are intersection and link. The combination of these elements constitutes the traffic network with link $z \in Z$ and junction

$j \in J$ which are defined geometrically exactly. Each signalized junction j has its own sets of incoming I_j and outgoing O_j links. Figure 1. shows the coherence (link z) of two neighboring intersections (M , N) in the transportation network where $z \in O_M$ and $z \in I_N$. The dynamic of link z is described by the conservation equation:

$$x_z(k+1) = x_z(k) + T[q_z(k) - h_z(k) + r_z(k) - s_z(k)] \quad (1)$$

where $x_z(k)$ measures the number of vehicles within link z , practically the length of queue, at time kT . $q_z(k)$ and $h_z(k)$ are the inflow and outflow, $r_z(k)$ and $s_z(k)$ are the demand and the exit flow during the sample period $[kT, (k+1)T]$. T is the control interval and $k = 0, 1, \dots$ is the discrete time index. For simplicity we assume henceforth that the cycle times are equal for each junction $j \in J$, namely $T_{c,j} = T_c$. Moreover T is also equal to T_c . $r_z(k)$ and $s_z(k)$ represent typically the fluctuation between a parking lot and link z or the effects of any non-controlled intersection between M and N . These disturbing flows can be considered as known perturbations if they can be well measured or estimated. In case of unknown disturbances robust control system is needed.

Equation (1) is linear scalar equation for the portrayal of vehicles movement of a given link. But if we wish to define a whole traffic network each link has to be described by its conservation equation and what is more the equations needs to be interconnected. At this point we can change for state space representation which means the appearance of the state and control input vectors together with the coefficient system matrices. The general discrete LTI state space representation is the following:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ed(k) \\ y(k) &= Cx(k) \end{aligned} \quad (2)$$

Using Equation (2), it is possible to describe the dynamics of an arbitrary urban traffic network (see Fig. 2 as an example).

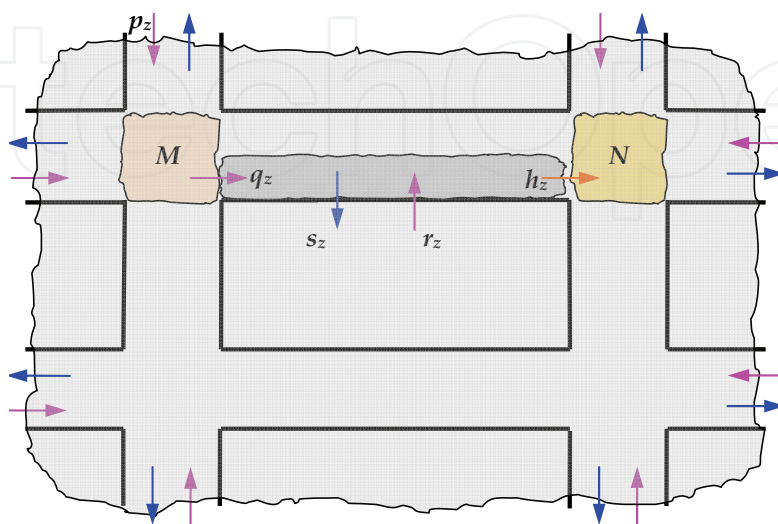


Fig. 2. Dynamics in the urban traffic network

The physical meaning of matrices and vectors is elementary to understand the model. The state equation form can be achieved using all conservation equations, arranging them in one linear matrix equality. In our case the state matrix A is practically considered as an identity matrix. The elements of the state vector $x(k)$ represent the number of vehicles of each controlled link. The second term of the state equation is the product of input matrix B and control input u . Vector u contains the green times of all stages. Their numerical values are the results of a corresponding controller at each cycle. Naturally the number of states is equal to the number of controlled links in the network. The product $Bu(k)$ is arising from the part $T[q_z(k) - h_z(k)]$ of Equation (1) which means the difference of the inflow and the outflow of a link during the control interval. $q_z(k)$ and $h_z(k)$ are directly related to control input (green time), saturation flow (S) and turning rate (t) in a signalized network. To understand the construction of B the parameters S and t have to be discussed. Saturation flow represents the outflow capacity of link $z \in Z$ during its green time. A standard value for saturation flow is $S = 0.5[\text{veh/sec}]$ which is considered constant in practice. Turning rate represents the distribution of turnings of vehicles from link $z \in O_j$ to links $w \in I_N$. These parameters are defined by the geometry and the rights of way in the traffic network and assumed to be known and constant or time varying. Then matrix $B = [b_{ij}]$ can be constructed by the appropriate allocation of the combinations of saturation flow and turning rates. The diagonal values of B are negative S_z as the product $S_z u_z(k)$ represents the outflow from link z . At the same time the inflow to the link z has to be also characterized. Therefore the products $S_z t_{w,z}$ are placed in matrix B such that $b_{ij} = S_z t_{w,z}$ when $i \neq j$. The parameters $t_{w,z}$ ($w \in I_M$) are the turning rates towards link z from the links that enter junction M . Hence the inflow is resulted from the appropriate matrix-vector multiplication for all z . In state space representation the third term $Ed(k)$ of Equation (2) represents an additive disturbance where $E = I$. $d(k)$ is composed of two type of data. On the one hand it is coming from the part $T[r_z(k) - s_z(k)]$ of Equation (1) where $r_z(k)$ and $s_z(k)$ are considered as measured disturbances. They reflect the difference of the demand and the exit flows of a link during the control interval. On the other hand there is demand $p_z(k)$ at the boundary of the traffic network (Figure 2.) which also has to be taken into consideration in the model. The traffic $p_z(k)$ intending to enter is a measurable value. Therefore it is simply added to the appropriate row of $d(k)$. To end the state space description of the urban traffic the measurement equation has to be mentioned. As each output inside of the network is a measured state (number of vehicles of the link $z \in Z$) the output equation is simplified to $y(k) = x(k)$ with $C = I$. Note that as the exit links of the network are not controlled they do not have to be confused with the outputs $y(k)$.

Finally, as three of the system matrices are identity matrix (discussed above) the general discrete LTI state space representation for urban traffic simplifies to the following form:

$$\begin{aligned} x(k+1) &= x(k) + Bu(k) + d(k) \\ y(k) &= x(k) \end{aligned} \quad (3)$$

3.2 Constraints of urban traffic control

As store-and-forward modeling technique tries to express the real dynamics and states of the urban traffic there are several constraints which have to be taken into account. The most essential constraints of the urban network are determined by the geometry. It is evident that the maximum number of vehicles is defined by the length of link between two junctions. Naturally the vehicles are considered as passenger car unit (PCU) resulting from appropriate transformation (Webster & Cobbe, 1966). Thus the states are subject to the constraints:

$$0 \leq x_z(k) \leq x_{z,max} \quad (4)$$

If we consider a network the use of the states constraints can contribute to avoid the oversaturation in the controlled traffic area. In a control scheme beside the state constraints one can define output limitations too. However in our case the states constraints are identically to the output constraints as $C = I$.

The control input is the next variable restricted by some constraints. The first constraint on u is the interval of seconds of green time:

$$u_{z,min} \leq u_z(k) \leq u_{z,max} \quad (5)$$

Depending on the system setting $u_{z,min}$ (for lack of vehicles on link z) can be zero. It means permanent red signal for the stage in the next control interval. The second control input constraint is represented by the linear combination of green times at junction $j \in J$. The sum of the green times has to be lower as $T_{j,max}$:

$$\sum_{z=1}^{O_j} u_z(k) \leq T_{j,max}, \quad j = 1, 2, \dots, J, \quad (6)$$

where O_j is the number of stages at junction j , $T_{j,max} = T - L_j$ (L_j is the fixed lost time resulted from the geometry of junction j), and J is the number of controlled intersections.

4. Simulation environment

In the previous sections traffic modeling was introduced which can be used in control design. Moreover the simulation environment has to be discussed similarly as all the methods presented in this chapter were simulated and tested. For simulation we used traffic simulator (VISSIM, 2010), numerical computing software (MATLAB, 2010) and C++ programming language.

VISSIM is a microscopic traffic simulation software for analyzing traffic operations. It is able to simulate network consisting of several intersections and allow the use of external control algorithm in the control processes. These properties make it suitable to use this software by reason of the several junctions and the control algorithms written in MATLAB. VISSIM uses a so-called psycho-physical driver behavior model based on the car-following model of (Wiedemann, 1974). The model describes all the cars found in the system. The vehicles are defined by both physical and psychical parameters (origin, destination, speed, driver

behavior, vehicle type, etc.). The VISSIM simulation is based on an iteration process of acceleration and deceleration.

The communication does not work directly between MATLAB and VISSIM as the simulation can only be accessed via Component Object Model (COM) interface (Roca, 2005). To control the communication a C++ application has to be created. The created C++ program manages the simulation process and controls the data transfer between the software (Figure 3.).

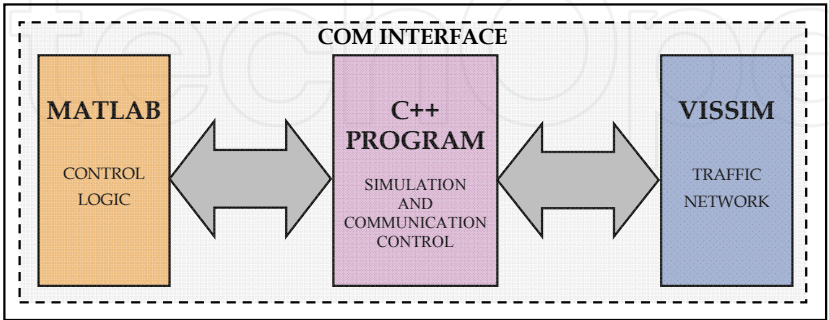


Fig. 3. The simulation process of the system model

5. MPC based urban traffic management

The aim of our research was to elaborate a control process related to network consisting of several junctions which perform the control of all the traffic lights in its sphere of action in a coordinated way depending on the traffic. The controller must be able to dynamically make the traffic signal set of the intersections. From the point of view of realization, this means that before every period a new traffic sign must be generated regarding all the traffic lights, in harmony with the present traffic. To solve the above, MPC technology was chosen since it is able to take all the constraints into consideration in course of the control input setting. To show the efficiency of MPC the control design was tested simulating a real-word traffic network.

5.1 The MPC cost function

The control objective is the minimization and balancing of the numbers of vehicles within the streets of the controlled network. This control objective is approached through the appropriate manipulation of the green splits at urban signalized junctions, assuming given cycle times and offsets. By employing the predictive control model, the dynamic determination (per cycle) of the traffic light’s period is possible either with the consideration of the natural constraints existing in the system introduced in Section 3.2.

The state space equation for MPC design can be given as follows:

$$\underbrace{\begin{bmatrix} x(k+1|k) \\ x(k+2|k) \\ \vdots \\ x(k+N|k) \end{bmatrix}}_{\tilde{x}(k+1)} = \underbrace{\begin{bmatrix} x(k)+d(k) \\ x(k)+2d(k) \\ \vdots \\ x(k)+Nd(k) \end{bmatrix}}_{c(k)} + \underbrace{\begin{bmatrix} B & 0 & \cdots & 0 \\ B & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B & B & \cdots & B \end{bmatrix}}_{\tilde{B}} \underbrace{\begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+N-1|k) \end{bmatrix}}_{g(k)}$$

(7)

where x , d , B and u are elements of Equation (3) already discussed. \tilde{x} is a hyper vector of the state vectors, representing the number of vehicles standing at each controlled link of

the intersections. c is a hyper vector combination of the previous state vector and d . The disturbance d is considered measured and constant during the horizons of k th step. Hence it is multiplied by the value of the current horizon. \tilde{B} is a lower triangular hyper matrix including the matrix B . g is a hyper vector of the control input vectors (green times), $k = 1, 2, \dots$ a is the discrete time index, and N is the length of the MPC horizon.

The MPC algorithm needs the current values of the states at each control interval which means the exact knowledge of the numbers of vehicle. However the states can not be directly measured only estimated using appropriate measurement system (e.g. loop detectors) and estimation algorithm. A possible realization for state estimation was published in paper of Vigos et al. (2007) which is based on the well-known Kalman Filter algorithm (Welch & Bishop, 1995). The estimation error is neglected in the paper.

The elements of B are the combinations of turning rates and saturation flow as discussed in Section 3.1. Saturation flow is not measurable hence a standard value is determined ($S = 0.5 [\text{veh/sec}]$). Usually the values of turning rates are also considered constant. Nevertheless, in practice the turnings vary around the nominal rates. Thus a continuous estimation may be applied to ameliorate the MPC algorithm. A possible way to estimate turning rates is to use a finite back stepped state observer, e.g. Moving Horizon Estimation (MHE) method (Kulcsár et al., 2005).

Several choices of the objective function in the optimization literature have been reported. In this chapter we consider the following quadratic cost function characterized by the weighted system states and control inputs:

$$J(k) = \frac{1}{2} \{ \tilde{x}^T(k) Q \tilde{x}(k) + g^T(k) R g(k) \} \rightarrow \min \quad (8)$$

where $Q > 0$ and $R > 0$ are scalar weighting matrices. Q and R have appropriately chosen tuning parameters in their diagonals. The weightings reflect that the control input variation is lightly punished compared to the state variation. The selection of the appropriate weightings is important, because this could influence (especially the end-point weight) the stability of the closed loop (Kwon & Pearson, 1978). To solve this minimization problem several mathematical software can be applied which provide built-in function for quadratic constrained optimization. The solution of optimization problem (8) leads to the minimization of the vehicle queues waiting for crossing intersections. The control input green time is defined corresponding to the states of intersection branches representing a fully adaptive traffic management.

Different stability proofs exist for receding horizon control algorithms. Maciejowski (2002), Rawlings & Muske (1993) or Mayne et al. (2000) offer different methodological approaches. However the urban traffic is a special case. It is ensured that the system will not turn instable because of the hard physical constraints coming from the network geometry. Accordingly, there is a natural saturation in the system. The states can never grow boundlessly. The instability can appear only if there is an oversaturation in the network. To solve this problem we intend to apply the results of the invariant set theory (Blanchini & Miani, 2007) in the future. It is also has to be noted that if we choose a traffic area to control we do not deal with the traffic outside of the boundary of the network. Obviously the sphere of control action is also an important question in traffic management.

5.2 Test network for simulation

To test MPC technology in urban traffic management we choose a real-world test area situated in the 10th district of Budapest. The test network includes seven neighboring intersections (Figure 4.).

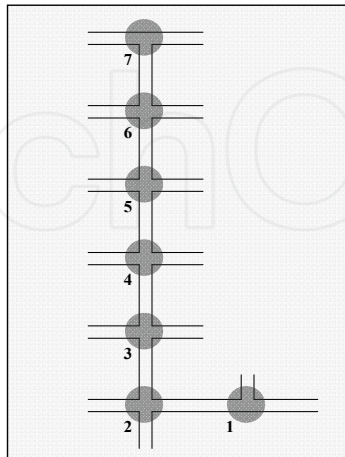


Fig. 4. Schematic representation of the test network consisting of seven junctions

The dimension of the system is 36 which means that we intend to control 36 links. This area is suitable for testing our new control system since the included road stretches have a heavy traffic volume in rush hours. The current traffic management system is offline. The seven junctions are controlled individually. Three of them use fix time signal plan. In the other four intersections detectors help the controllers. They can slightly modify their fix programs. The current control is effective but only in case of normal traffic flow. If the volume of vehicles increases extremely, the system cannot manage the situation and traffic becomes congested before the stop lines. The biggest problem is that the controllers work locally and independently. Our new control design, however, takes the seven junctions into consideration as a real network.

As the MPC cost function (8) represents a quadratic optimization problem the control input was calculated using the built-in *quadprog* function of MATLAB.

5.3 Simulation results

To prove the applicability of the MPC based control design it was compared with the current control system of the test network, which is a partly adaptive control strategy.

The same input traffic volumes were set for both simulations. We used volume data for which the traffic lights were originally designed. The simulation provided similar results for both strategies as we expected. This means the current system is correctly designed, and manages non-extreme traffic flow with good results.

To test the effectiveness of the two systems in case of heavier traffic we generated more intensive traffic flow during the simulation. The original input volumes were increased by 10% in the network. This simulation showed different results to the previous case. The current system could manage the traffic less efficiently compared with the MPC based control system. The simulation time was 1 hour long. The results are presented in Table 1. All important traffic parameters changed in a right way. The new system can provide a very effective control in the test network.

Parameter	OLD STRATEGY	MPC based strategy	Variation
Total travel time per vehicle [sec]	114	96	↓ 16%
Average speed [km/h]	20.6	24.9	↑ 21%
Average delay time per vehicle [sec]	68	56	↓ 18%
Average number of stops per vehicles	3.8	3.1	↓ 18%

Table 1. Average simulation results of the test network

At the same time these simulations were run in a reduced environment. We diminished the number of junctions in the test network from seven to four. Namely the traffic lights at junctions 4., 5., 6. (see Figure 4.) work totally offline. The capacities of these locations increased apparently. So only the junctions 1., 2., 3., and 4. were kept in order to focus on the comparison of the two adaptive strategies.

Parameter	Old strategy	MPC based strategy	Variation
Total travel time per vehicle [sec]	105	96	↓ 9%
Average speed [km/h]	20.5	23.5	↑ 15%
Average delay time per vehicle [sec]	64	52	↓ 19%
Average number of stops per vehicles	1.2	1.2	0%

Table 2. Average simulation results of the test network with design input volumes

Parameter	Old strategy	MPC based strategy	Variation
Total travel time per vehicle [sec]	110	96	↓ 13%
Average speed [km/h]	18.4	23.6	↑ 28%
Average delay time per vehicle [sec]	71	52	↓ 27%
Average number of stops per vehicles	1.5	1.2	↓ 20%

Table 3. Average simulation results of the test network with 10% augmentation of the design input volumes

Alike above, the behavior of the reduced network was analyzed with normal and heavier input traffic volumes. The results ameliorated in both cases (see Table 2. and 3.). The simulation time was 2 hours long.

The aim of the MPC based control is the minimization of the number of vehicles waiting at the stop line. The current system cannot adapt to the increased volume. The average queue length grew strongly during the simulations. However, the MPC strategy is able to manage heavier traffic situations real-time. Figure 5. represents the effectiveness of our system. It shows the variation of average queue lengths in the network.

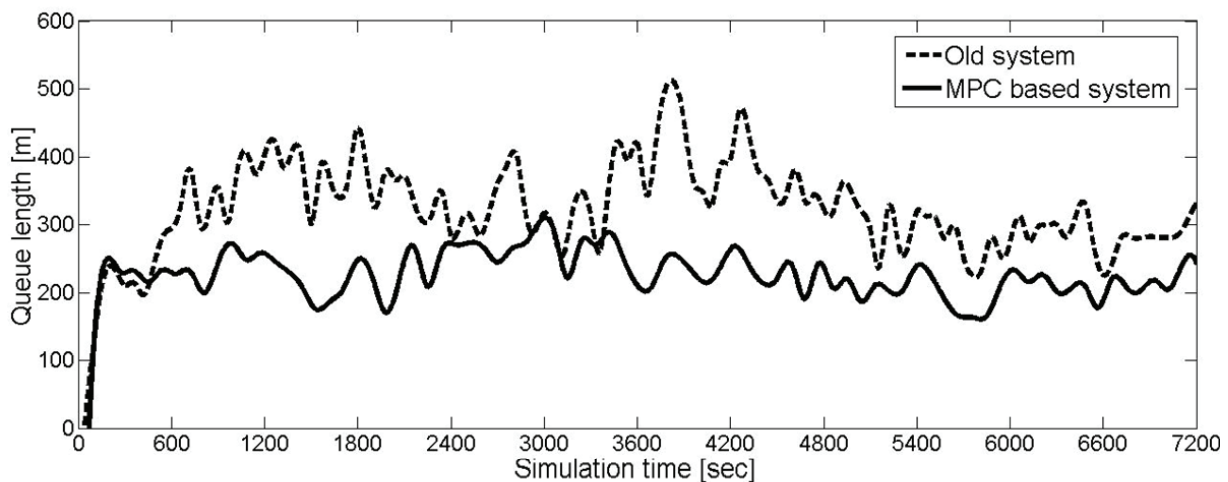


Fig. 5. The variation of average queue lengths in the two different control cases

6. Distributed traffic management system based on MPC

The classical scheme for adaptive road traffic management structure is based on control center which processes and computes all signal control for the network. Another method for the control system architecture is the decentralized and distributed control scheme. This approach has numerous economical and technological advantages.

In this section we present a distributed control system scheme for urban road traffic management. The control algorithm is based on MPC involving Jacobi iteration algorithm to solve constrained and nonlinear programming problem. The distributed control design was also simulated and tested.

6.1 The MPC cost function

We refer to the results of Section 5.1. Substituting $\tilde{x}(k)$ and $g(k)$ in Equation (8) one arrives to:

$$J(k) = \frac{1}{2} g^T (q \tilde{B}^T \tilde{B} + r I) g + q c^T \tilde{B} g + \frac{1}{2} q c^T c = \frac{1}{2} g^T \Phi g + \beta^T g + \gamma \quad (9)$$

where q and r are constants coming from the diagonal of the scalar matrices Q and R . As γ is a constant term, finally one has the objective function to minimize:

$$J(k) = \frac{1}{2} g^T(k) \Phi g(k) + \beta^T(k) g(k) \rightarrow \min \quad (10)$$

where Φ is constant matrix as it contains the combination of constant turning rates, saturation rates and fixed tuning parameters. At the same time β contains varying values coming from the current dynamics of the traffic area.

6.2 Multivariable nonlinear programming to solve MPC problem

The solution of the MPC cost function (10) represents a multivariable nonlinear problem subject to linear constraints. It formulates a standard quadratic optimization problem (Bertsekas & Tsitsiklis, 1997):

$$J(k) = \frac{1}{2} g^T \Phi g + \beta^T g \rightarrow \min$$

$$\text{s. t. } Fg - h \leq 0 \quad (11)$$

where matrix inequality $Fg \leq h$ incorporates the constraints (4), (5) and (6) already discussed in Section 3.1.

If Φ is a positive semi definite matrix, (11) gives a convex optimization problem (Boyd & Vanderberghe, 2004). Otherwise one has to use the singular value decomposition method to Φ which results a convex problem. This means a linear transformation to the original problem (11).

Using the duality theory (Bertsekas & Tsitsiklis, 1997) the primal problem can be formulated into Lagrange dual standard form. The basic idea in Lagrangian duality is to take the constraints into account by augmenting the objective function with a weighted sum of the constraint functions. We define the Lagrangian associated with the problem as:

$$L(g, \lambda) = J(k) + \lambda^T (Fg - h) \quad (12)$$

We refer to λ_i as the Lagrange multiplier associated with the i th inequality constraint of (11). The dual function is defined as the minimum value of the Lagrangian function. This can be easily calculated by setting gradient of Lagrangian to zero (Boyd & Vanderberghe, 2004). This yields an optimal green time vector (16) which minimizes the primal problem. Hence one arrives to the dual of the quadratic programming problem:

$$J_{DUAL}(k) = \frac{1}{2} \lambda^T P \lambda + w^T \lambda \rightarrow \min$$

$$\text{s. t. } \lambda \geq 0 \quad (13)$$

where P and w are coming from the original problem:

$$P = F\Phi^{-1}F^T \quad (14)$$

$$w = F\Phi^{-1}\beta + h \quad (15)$$

It is shown that if λ^* provides optimal solution for the $J_{DUAL}(k)$ problem then

$$g^* = -\Phi^{-1}(\beta + F^T \lambda^*) \quad (16)$$

gives also an optimal solution for the primal problem (Rockafellar, 1970).

The dual problem has a simple constraint set compared with the primal problem's constraints. Hence expression (13) represents a standard minimization problem over nonnegative orthant.

A very efficient method, the Jacobi iteration was found to solve the optimization problem. Since Φ is a positive semi definite matrix the j th diagonal element of P , given by

$$p_{jj} = f_j^T \Phi^{-1} f_j \quad (17)$$

is positive. This means that for every j the dual cost function is strictly convex along the j th coordinate. Therefore the strict convexity is satisfied and it is possible to use the nonlinear Jacobi algorithm. Because the dual objective function is also quadratic the iteration can be written explicitly. Taking into account the form of the first partial derivative of the dual cost

$$w_j + \sum_{k=1}^n p_{jk} \lambda_k \quad (18)$$

the method is given by:

$$\lambda_j(t+1) = \max \left\{ 0, \lambda_j(t) - \frac{\kappa}{p_{jj}} \left(w_j + \sum_{k=1}^n p_{jk} \lambda_k(t) \right) \right\}, \quad j=1, \dots, n \quad (19)$$

Where $\kappa > 0$ is the stepsize parameter which should be chosen sufficiently small and some experimentation may be needed to obtain the appropriate range for κ .

The importance of this method, over its efficiency, is the ability to satisfy the positivity since equation (19) excludes negative solution for λ . Thus, during the MPC control process at each (k th) step the optimal green times can be directly calculated from equation (16) after solving the problem (11).

6.3 Realization of MPC based distributed traffic management system

The economical and technological innovation of the above described control method is represented by the state-of-the-art control design and the optional decentralized realization at the same time.

Generally, the architectures of traffic control systems can be central, decentralized, or mixed. The central management architecture is a frequent strategy based on a central processor which controls all signal controllers in the transportation network. Decentralized and mixed control systems are not so common applications yet. However they have many advantages and represent a new way in traffic control technology. Decentralized management systems carry a higher performance since they can distribute their computations between the traffic controllers. As well as they represent a higher operation safety because of their structural redundancy. Some of these distributed realizations are for example SCATS (Wolshon & Taylor, 1999) or Utopia (UTOPIA, 2010).

The distributed technology can be used in any road traffic network which is equipped with adequate signal controllers and detectors, as well as communication between controllers is also required.

Since the solution of the Jacobi algorithm (19) is an iteration process the computers can distribute their calculations during the operation cycle. Therefore it is suitable for the distributed realization of the MPC problem. Considering a large traffic network the following practical system realization can be applied. Firstly we define the nodes represented by the red cubes on Figure 6. The nodes are the head traffic controllers which participate in the resolution procedure. Every node covers a few intersections (traffic

controllers) which do not participate in the computation. The distributed control network is represented by Figure 6.

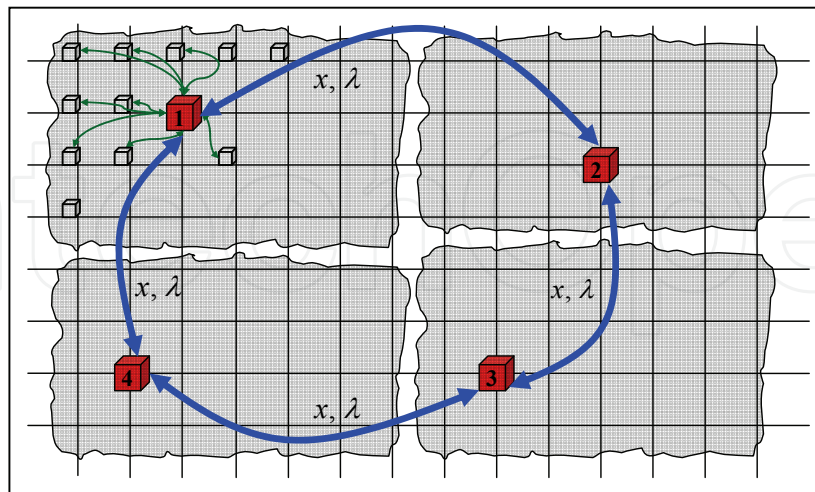


Fig. 6. Distributed MPC control in urban traffic network

The distributed computation is executed during the operation cycle as follows:

1. Communication: At the end of the k th cycle all traffic controllers send their measurements (number of vehicles) to their node.
2. Communication: The head controllers share the measurement data with the other nodes.
3. Calculation (which is not the third step practically since it can be started parallel with step 1.): Node 1. starts iteration procedure. After some predefined iteration steps it forwards their computational results to the next node and so on. The transmitted data is the currently calculated vector λ_s where $s = 1, 2, \dots, \kappa$ (κ represents the final iteration step number specified previously).
4. Communication: When the last node finishes the computation (which means practically that $s = \kappa$) it shares the optimal result (λ^*) with the other nodes.
5. Calculation: The head controllers calculate their final calculation. Using Equation (16) the nodes do not need to execute the whole multiplication. But only the specified part of g^* which contains the optimal green times of their traffic controllers.
6. Communication: Finally the nodes pass the optimal green times to every traffic controllers for the next $(k+1)$ cycle.

If one wishes to control small traffic network with a few intersections the distributed solution is not certainly required. The calculation of one Jacobi iteration step means simple multiplications and additions of scalars. In case of few states (number of the controlled links) a single controller's performance is sufficient to compute all signal sets of the network. Thus the system is working with redundancy which can be very useful at the same time. The controllers can continuously check their operation comparing their computation results. On the other hand, if one of the signal controllers fails in the network the system can go on with safe functioning.

However with the growth of the number of the system states the computational demand increases quadratically. Therefore larger network requires the distributed solution of the

MPC control. Certainly the solution method is also largely depends on the performance of the actual signal controllers and the communication system.

6.4 Simulation of MPC based distributed control algorithm

To verify the designed control system scheme a closed loop simulation environment was created (Section 4.). The traffic network used for the simulation is equivalent with the one applied in Section 5.2.

As discussed before the appropriate setting for the stepsize parameter κ requires some practical experimentation. This value strongly influences the performance of the calculation. Convergence can be shown when $\kappa = n^{-1}$. However this value may lead to an unnecessarily slow rate of convergence for some problems.

The values in Table 4. shows the variation of the number of steps to achieve convergence. In case of our test network the smallest value with convergence was $\kappa = n^{-0.0525}$.

κ	Number of steps
n^{-1}	6000
$n^{-0.5}$	1000
$n^{-0.1}$	200
$n^{-0.0525}$	150

Table 4. The variation of the number of steps to achieve convergence

We also compared the computation times of the applied methods. Using the *quadprog* function of MATLAB the computation time was about 20 seconds. Conversely the Jacobi algorithm required less than 1 second on average which means 20 times faster calculation. It has to be noted that the Jacobi algorithm was not tested in a distributed way. However even with some communication time the Jacobi iteration is more efficient. On the one hand in our test network the number of states was quite few. The distributed solution is not needed. On the other hand the distributed realization is highly dependent on the current system configuration (measurement accuracy, communication speed, etc.).

6.5 Vehicle priority management in MPC based urban control

The design of an adaptive traffic control system comes with the desirable demand to incorporate vehicle priority management as well. Therefore an additional feature of the designed system is the ability to manage priority.

The scope of the priority management has to be specified as some special vehicle classes (e.g. emergency vehicles) have top-level priority. Therefore they do not need any help from traffic lights to cross the intersections anytime. Our control deals vehicles which are favored compared to the others but not by all means. Vehicles of the public transport are typically of this sort. However one may differentiate the levels of importance even between public vehicles (e.g. an overland bus compared to a local bus).

To operate such system these vehicles have to be able to communicate with the traffic controllers. If a preferred vehicle arrives to any junction of the network it may be automatically indicated by the traffic controller through radio frequency. Its stage can be handled with priority getting maximum green time as possible in every cycle until the

vehicle will not leave the intersection. It means practically that the cost function is dynamically modified by the system weights depending of the presence of any preferred vehicles. Accordingly for the sake of immediate reaction the given junction falls out of the scope of the coordinated traffic control until the vehicle will not leave the intersection. However it can be considered as disturbance.

We refer to the original MPC cost function (8) where Q is a diagonal weighting matrix:

$$Q = \begin{bmatrix} q_1 & & & \\ & q_2 & & \\ & & \ddots & \\ & & & q_n \end{bmatrix} \quad (20)$$

Each diagonal element tunes a state (queue length of controlled links). If there is no preferred vehicle in the scope of control: $q_1 = q_2 = \dots = q_n$. By online modifying the weight q_i (according to the preferred vehicle's direction) one can assure priority. The measure of the modification of q_i depends on the current level of priority. In practice, the appropriate choice of the weights is an empirical process as it strongly depends on the junction's properties.

7. Future work: Robust MPC in urban traffic management

As future work we introduce the problem of robustness in urban traffic management. In Section 3.1 all disturbances in the state space model were considered as known (measured) values and all possible uncertainties were neglected. These assumptions were taken by practical reasons. However for more precise traffic modeling these factors can be involved in the control scheme determining upper and lower bounds of the uncertainties. This implies the use of a suitable robust control method as well.

The simplest approach to represent disturbances in the system is the bounded unknown external additive disturbance. In this case an additive term appears in the LTI state space model. This approach can deal with state disturbances. As a part of the Ph.D thesis of Löfberg (2003) a Minimax MPC is presented which can be eligible for traffic systems too.

Another possibility to model the uncertainties is the polytopic paradigm. The system matrices $A(k)$ and $B(k)$ of an LTV state space description can be defined by a prespecified polytopic set:

$$\Omega = Co\{[A_1 \ B_1], [A_2 \ B_2], \dots, [A_L \ B_L]\} \quad (21)$$

where Co devotes to the convex hull and L is the number of the vertices. Matrix $A(k)$ can be used to express uncertainties of the states. In practice it means for example parking places along the road or non-controlled junctions in the network which result unmeasured state variation. Matrix $B(k)$ can be used to represent uncertainties of the saturation flow rates which are also non-measurable parameters. For polytopic system Kothare et al. (1996) provide an efficient Minimax MPC solution which can be potentially applied in urban traffic management as well.

There is another factor which can be taken into consideration in robust traffic control. In Section 5.1 the demands (d) intending to enter the network were assumed constant and

measured disturbances. In effect they vary continuously. Therefore for fully exact solution varying demands should be considered in the MPC cost function.

8. Conclusion

This chapter introduced the aspects of MPC applied in urban traffic management. As the urban traffic is a complex system having special attributes the appropriate traffic model had to be discussed in details as well. At the same time MPC technology is suitable to control such complex system optimally and real-time. The main control aim was the optimal and coordinated control which can be satisfied. The applicability was demonstrated by several simulations. Furthermore a distributed technology was presented which can be very useful in practice particularly in large traffic network. As an additional feature of MPC based system we showed that an optional vehicle priority management can be easily implemented in the control design. Finally we introduced the possibility of the robust control in urban traffic which is a planned research scope in the future.

9. Acknowledgement

This work is connected to the scientific program of the "Development of quality-oriented and harmonized R+D+I strategy and functional model at BME" project. This project is supported by the New Hungary Development Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002) and by the Hungarian Scientific Research Fund (OTKA) through grant No. CNK 78168 and by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences which are gratefully acknowledged.

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Model Predictive Control

Edited by Tao Zheng

ISBN 978-953-307-102-2

Hard cover, 304 pages

Publisher Sciyo

Published online 18, August, 2010

Published in print edition August, 2010

Frontiers of Model Predictive Control Robust Model Predictive Control Nonlinear Model Predictive Control
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How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Tamás Tettamanti, Istvan Varga and Tamas Peni (2010). MPC in Urban Traffic Management, Model Predictive Control, Tao Zheng (Ed.), ISBN: 978-953-307-102-2, InTech, Available from:
<http://www.intechopen.com/books/model-predictive-control/mpc-in-urban-traffic-management>

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